Measurement of spectral function in the decay $\tau^- \to \pi^- \pi^0 \nu_\tau$

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Outline

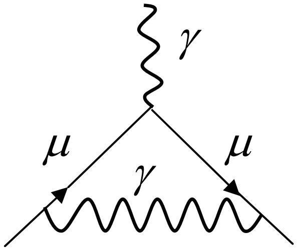
1. Motivation
   ~ Muon Anomalous Magnetic Moment $g_\mu - 2$ ~
2. Event selection
3. $\pi\pi^0$ mass spectrum (unfolding)
4. Evaluation of $a_\mu^{\pi\pi}$
5. result

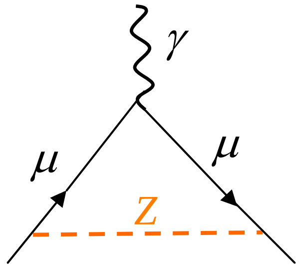
Motivation
~ Muon Anomalous Magnetic Moment $(g_\mu - 2)$ ~

Muon Anomalous Magnetic Moment: $a_\mu = \dfrac{g_\mu - 2}{2}$

- The prediction of Standard Model

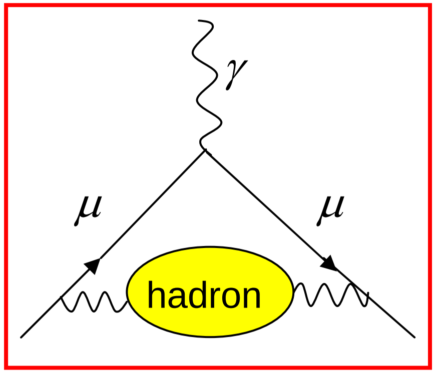
$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had} + a_\mu^{had,LBL}$$

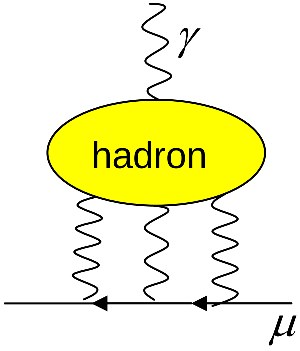




largest error from
Hadron vacuum polarization

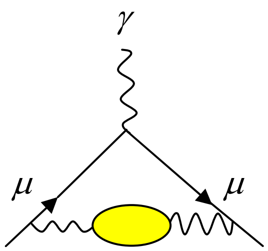
a_μ^{had}





a_μ^{had} and $\tau \to \pi\pi^0\nu_\tau$ decay

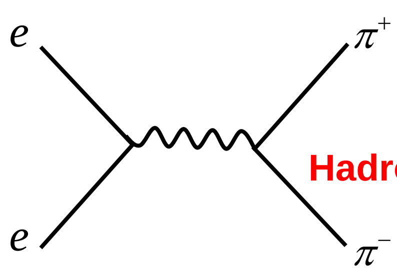
the contribution of hadrom Vacuum polarization (a_μ^{had})



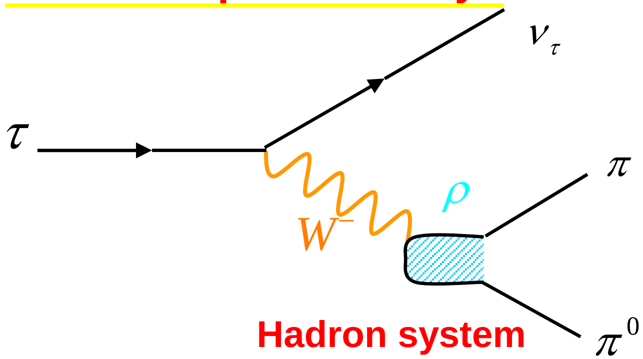
difficult to obtain from first principle !

obtained from Experimental Data.

Cross section of $e^+e^- \to \text{hadron}$



τ Semi-Leptonic decay



$\tau \to \pi\pi^0\nu_\tau$ decay is

useful to determine the term , a_μ^{had}

73% from 2π system

$a_\mu^{\pi\pi}$ and $\tau \to \pi\pi^0\nu_\tau$ decay

Hadron Vacuum polarization term from 2π system ($a_\mu^{\pi\pi}$)

$$a_\mu^{\pi\pi} = \frac{\alpha_{em}^2(0)}{\pi}\int_{4M_\pi^2}^{\infty} ds\, \frac{K(s)}{s} v^{\pi\pi^0}(s)$$

$s = M_{\pi\pi}^2$

K(s) is known function.

Spectral function

$$v^{\pi\pi^0}(s) = \frac{M_\tau^2}{6\pi|V_{ud}|^2 S_{EW}}\left[\left(1-\frac{s}{M_\tau^2}\right)\left(1+\frac{2s}{M_\tau^2}\right)\right]^{-1}\frac{B_{\pi\pi^0}}{B_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}$$

$\pi\pi^0$ mass square spectrum

$\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}$ is measured in this experiment.

Present status ; Muon Anomalous Magnetic Moment ($g_\mu - 2$)

- **Exp.** ･･･measured by BNL (g-2) experiment. $a_\mu^{\exp} = (11659203 \pm 8) \times 10^{-10}$ (2002.9)
- **Theoretical prediction** ･･･ new e^+e^- data (CMD-2) and τ data (ALEPH) (2003.1)
  - e^+e^- base

$$a_\mu^{SM} = \left(11659169.3 \pm 7.0(had) \pm 3.5(LBL) \pm 0.4(QED + EW)\right) \times 10^{-10}$$

$$a_\mu^{\exp} - a_\mu^{SM} = (33.7 \pm 11.2) \times 10^{-10} \quad \Rightarrow \quad \text{difference by } 3.0\sigma$$

  - τ base

$$a_\mu^{SM} = \left(11659193.6 \pm 5.9(had) \pm 3.5(LBL) \pm 0.4(QED + EW)\right) \times 10^{-10}$$

$$a_\mu^{\exp} - a_\mu^{SM} = (9.4 \pm 10.5) \times 10^{-10} \quad \Rightarrow \quad \text{agree within } 0.9\sigma$$

- Hadron Vacuum Polarization. term is different between e^+e^- and τ base predictions.
- Cross check is important !

Event selection

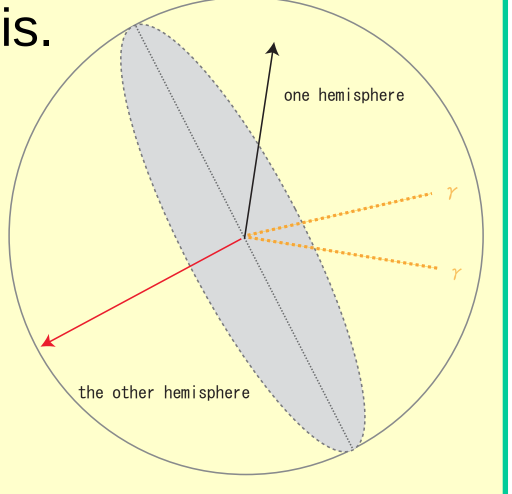
Data : $4.43 fb^{-1}$ accumulated from 2000.10 to 2000.12 at ***Belle***.

(corresponding to 4.0×10^{6} $\tau^{+}\tau^{-}$ production.)

$e^{+}e^{-}\rightarrow\tau^{+}\tau^{-}$ event selection

$e^{+}e^{-}\rightarrow\tau^{+}\tau^{-}$ event selection criteria

- Number of charged tracks：2 or 4
- All charge (ΔQ) = 0
- event vertex position : $|V_z| < 2.5$ cm , $|V_r| < 0.5$ cm
- Separate the event into 2 hemisphere by the event axis.

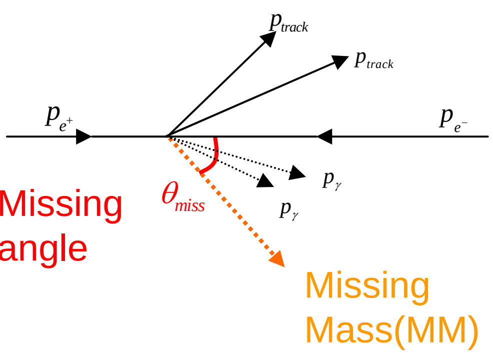


- Event axis direction : $35^{\circ} < \theta^{*} < 145^{\circ}$
- Back Ground rejection (next slide)
- Physics trigger

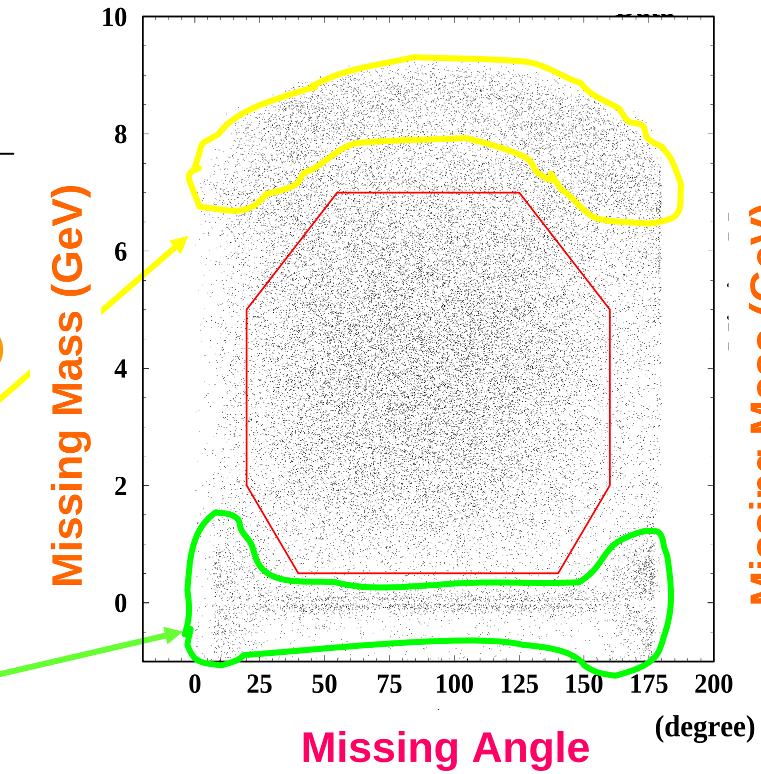
$e^+e^- \to \tau^+\tau^-$ event selection (Back ground rejection)

- **Bhabha , $e^+e^- \to \mu^+\mu^-(\gamma)$ and two photon rejection**
  - Missing mass and Missing angle cut (MM v.s. θ_{miss} plot)

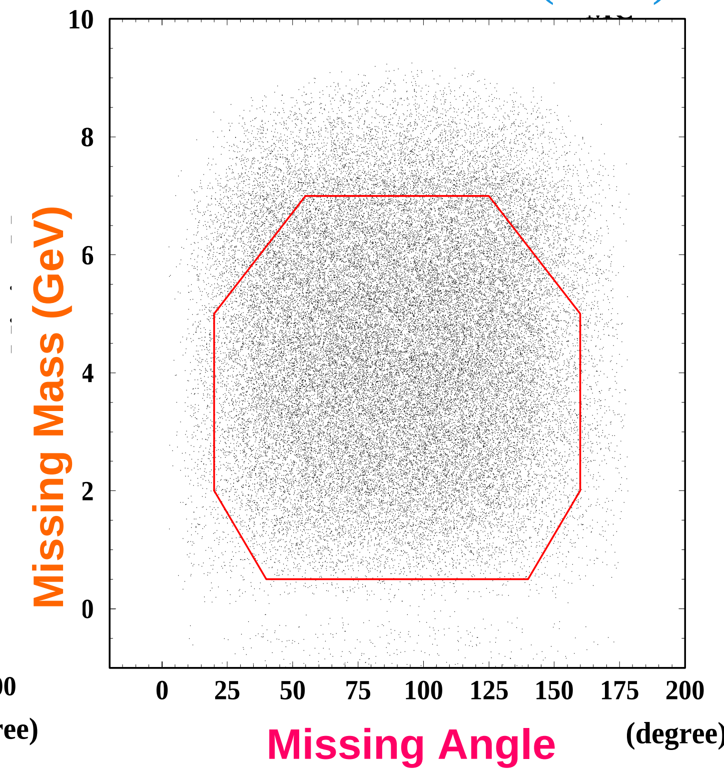
$$MM^2 = \left(p_{e^+} + p_{e^-} - \sum p_{track} - \sum p_\gamma\right)^2$$



Data



MC $(\tau^+\tau^-)$



- **Hadron($e^+e^- \to q\bar{q}$) rejection**
  - Reject high-multiplicity event ($X_{part} \equiv (n_{track} + n_\gamma)_{one} \times (n_{track} + n_\gamma)_{other} \leq 25$)

➡ **about 1,300,000 $e^+e^- \to \tau^+\tau^-$ events are remained.**

$\tau \to \pi\pi^0\nu_\tau$ Event selection

$\tau \to \pi\pi^0\nu_\tau$ selection criteria

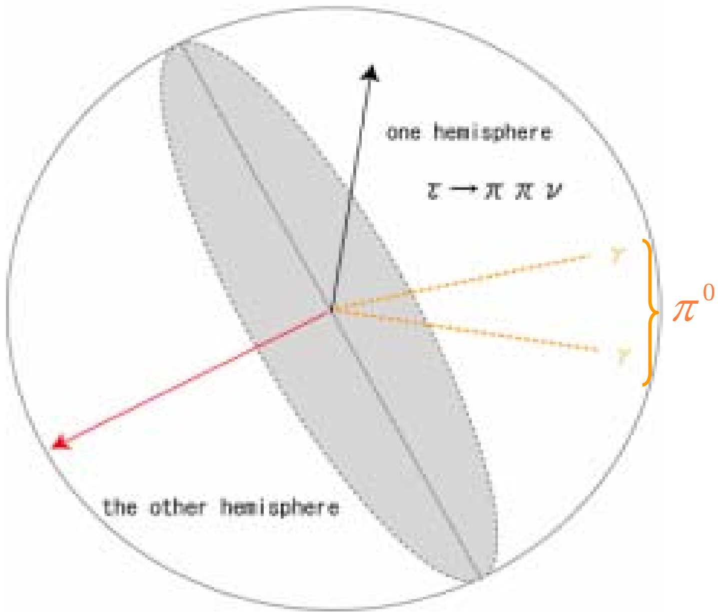
- one charged track in hemisphere.
- one π^0 in the hemisphere.

  gamma condition : gamma-like shower shape

  : $E_\gamma > 0.08$GeV

- veto the additional gamma

  (with high momentum (more than 200 MeV/c))



∗We do $\tau \to \pi\pi^0\nu_\tau$ analysis each hemispheres.

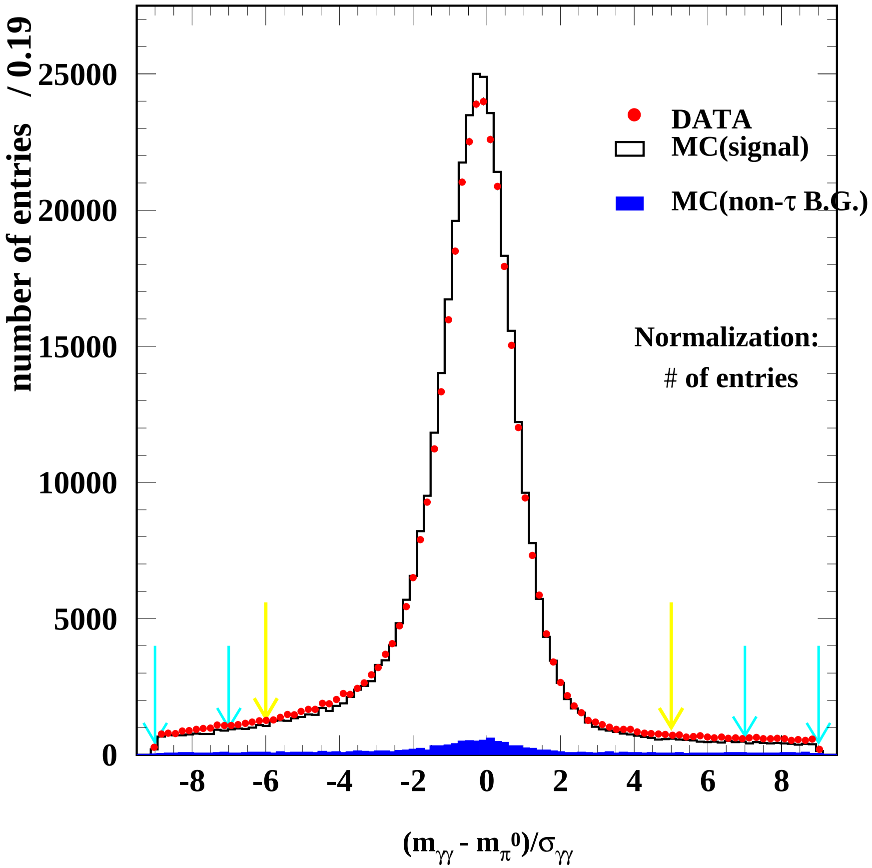
π^0 signal

$$S_{\gamma\gamma} \equiv \frac{(m_{\gamma\gamma} - m_{\pi^0})}{\sigma_{\gamma\gamma}}$$

m_{π^0} : π^0 **Mass** (134.98_{MeV})

$m_{\gamma\gamma}$: $\gamma\gamma$ **invariant mass distribution**

$\sigma_{\gamma\gamma}$: **resolution of** $m_{\gamma\gamma}$



Signal region

$-6 < S_{\gamma\gamma} < 5$

Side-band region

right: $-9 < S_{\gamma\gamma} < -7$

left: $7 < S_{\gamma\gamma} < 9$

Side-band region are used for estimation of π^0 BG in signal region.

$$N_{signal} = N_{sig}^{total} - \left((N_{side}^{left} + N_{side}^{right}) \times \frac{11}{4} \right)$$

320,000 $\tau \to \pi\pi^0\nu_\tau$ events.

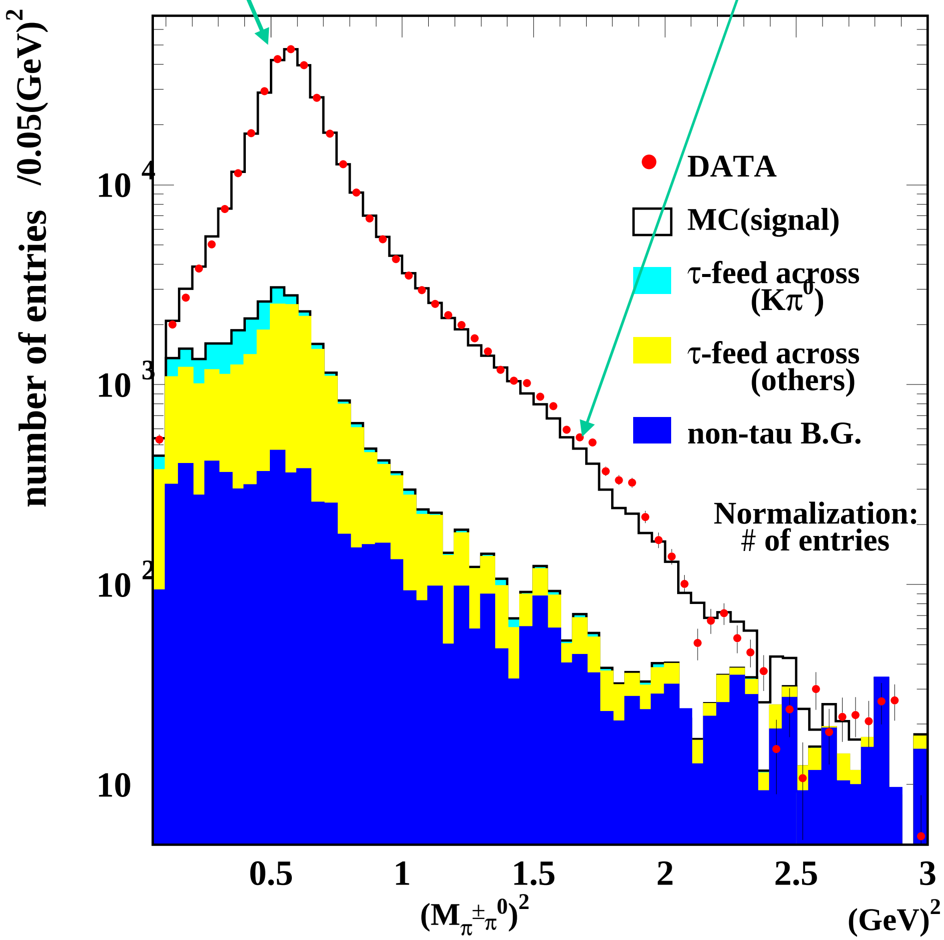
$\pi\pi^0$ mass spectraum

Clear peak of $\rho(770)$.

$\tau \to \rho\nu_\tau$ is dominant.

$\rho(1450)$ **peak.**

$\tau \to \rho'\nu_\tau$ is also included .



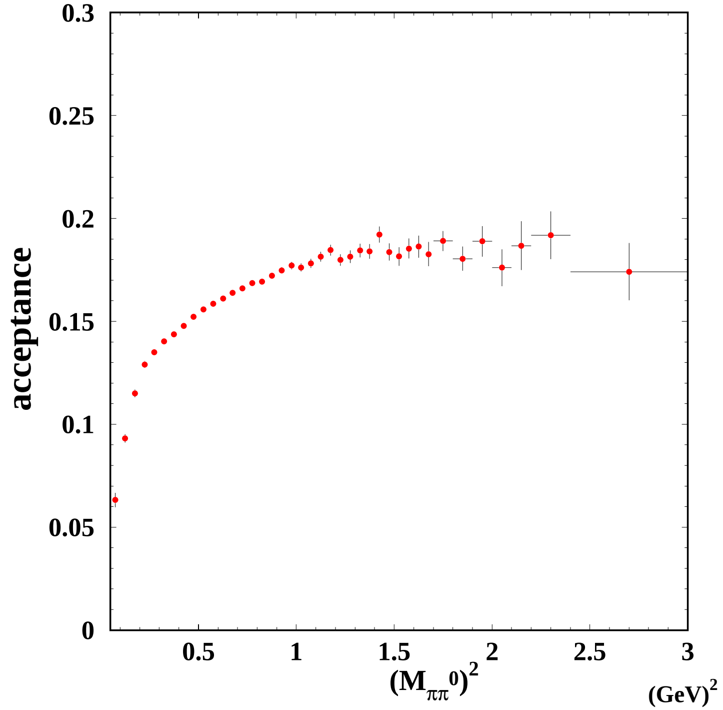
B.G. fraction

source	B.G. fraction
non-τ B.G.( )	
$ee \to q\bar{q}$	2.3 ± 0.06 %
Two photon	0.14 ± 0.01 %
Feed across B.G.	
$h \geq 2\pi^0\nu$ ( )	5.43 ± 0.08 %
$K\pi^0$ ( )	1.74 ± 0.09 %
Total	6.1 ± 0.10 %

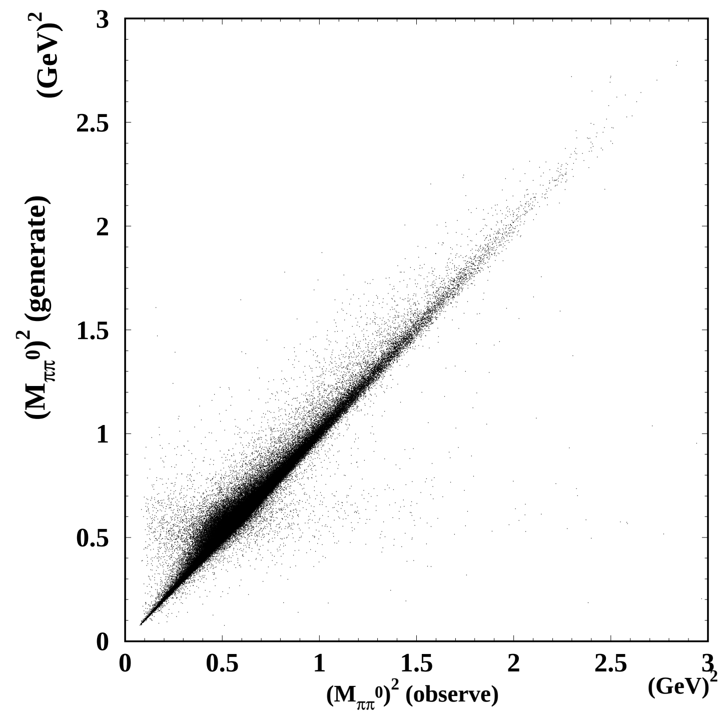
Unfolding

Acceptance and bin-by-bin migration effects are corrected via **S**ingular-**V**alue-**D**ecomposition method.

acceptance



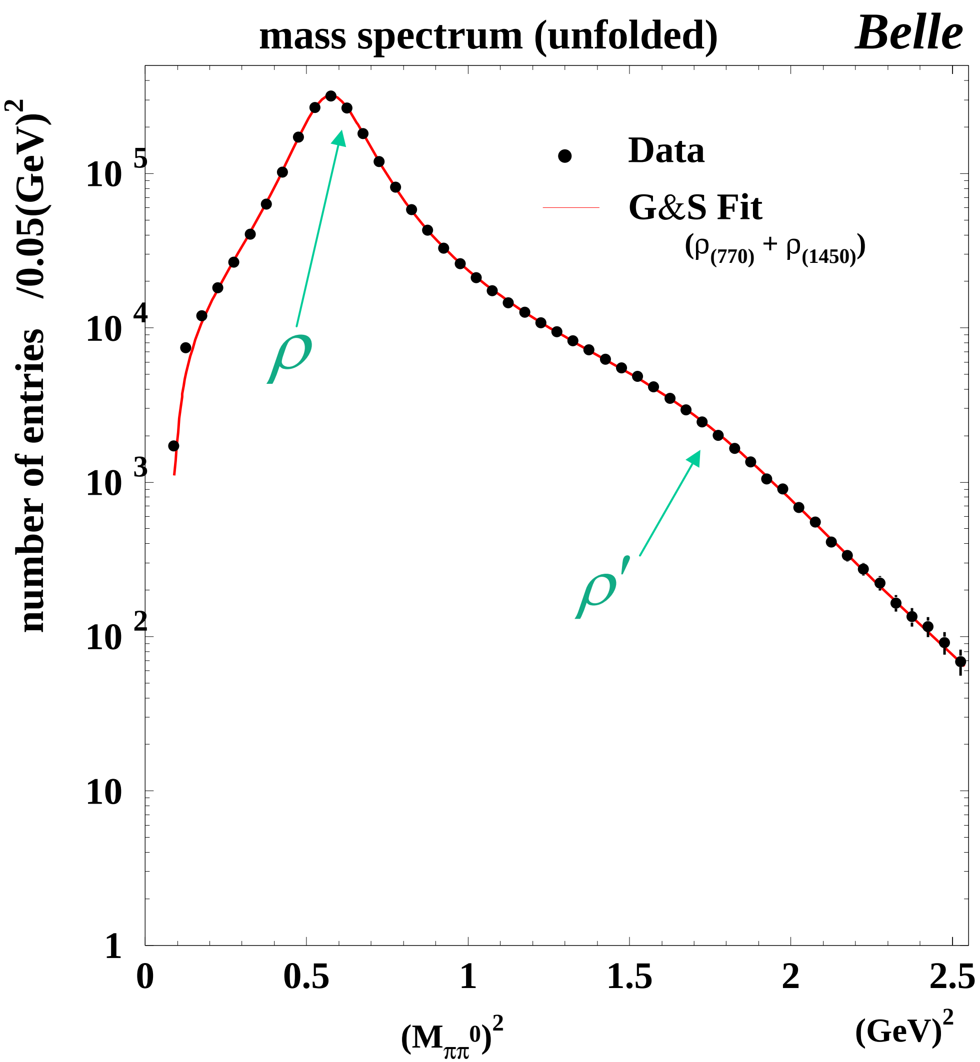
$M^2_{\pi\pi^0}\big|_{gen}$ V.S. $M^2_{\pi\pi^0}\big|_{obs}$



Acceptance include both the tau-pair and pipi0 selection.

Mass square resolution : 0.03 GeV^2

Unfolded mass spectrum



Red line :

Breit Wigner fitting function

(ρ and ρ' are included.)

Breit Wigner fitting form

$$\frac{dN}{ds} = A\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right)\cdot v(s)$$

$$v(s) = \frac{1}{12}\left|F_\pi(s)\right|^2 \beta_\pi^3$$

$$F_\pi(s) = \frac{1}{1+\beta e^{i\phi}}\left(BW_\rho + \beta e^{i\phi}\cdot BW_{\rho'}\right)$$

$$BW_\rho^{GS} = \frac{M_\rho^2 + d\cdot\Gamma_\rho\cdot M_\rho}{(M_\rho^2 - s) + f(s) - i\sqrt{s}\cdot\Gamma_\rho(s)}$$

$\frac{dN}{ds}$; $M_{\pi\pi^0}^2$ mass distribution

$s = M_{\pi\pi^0}^2$

$v(s)$; Spectral function

free parameter

$A, M_\rho, \Gamma_\rho, M_{\rho'}, \Gamma_{\rho'}, \beta, \phi$

Gounaris and Sakurai (G&S) Model

GS model is known that it can fit wilder mass region that the commonly used BW.

fit result and compare with previous Experiments

Fit Parameter	***Belle***	***CLEO***	***ALEPH***
M_ρ (MeV)	773.9 ± 0.4	775.3 ± 0.5	776.4 ± 0.9
Γ_ρ (MeV)	152.4 ± 0.7	150.5 ± 1.1	150.5 ± 1.6
$M_{\rho'}$ (MeV)	1398 ± 21	1365 ± 7	1400 ± 16
$\Gamma_{\rho'}$ (MeV)	450 ± 40	356 ± 26	$\equiv 310$ (fixed)
β	0.085 ± 0.010	-0.108 ± 0.007	-0.077 ± 0.008
ϕ (degree)	181.0 ± 6.2	$\equiv 180.0$ (fixed)	$\equiv 180.0$ (fixed)
$\chi^2 / d.o.f$	$35.6/42$	$26.8/24$	$54/65$

- ρ parameters : good agreement with previous Exp.
- ρ' parameters : ***Belle*** results are most precise.

Evaluation of $a_\mu^{\pi\pi}$

$$a_\mu^{\pi\pi} = \frac{\alpha_{em}^2(0)}{\pi} \int_{4M_\pi^2}^{\infty} ds \frac{K(s)}{s} v^{\pi\pi^0}(s)$$

$$v^{\pi\pi^0}(s) = \frac{M_\tau^2}{6\pi |V_{ud}|^2 S_{EW}} \left[\left(1 - \frac{s}{M_\tau^2}\right)\left(1 + \frac{2s}{M_\tau^2}\right) \right]^{-1} \frac{B_{\pi\pi^0}}{B_e} \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds}$$

- M_τ (tau mass)
- S_{EW} (Electro-Weak correction factor)
- $|V_{ud}|$ (element of CKM matrix)
- B_e (Br. of $\tau \to e\bar{\nu}_e \nu$)
- $B_{\pi\pi^0}$ (Br. of $\tau \to \pi\pi^0 \nu$)

Systematic error (Ⅰ)

External systematics ~Normalization factors~

source	value	$\Delta a_\mu^{\pi\pi}$ (10^{-10})
S_{EW}	1.0199 ± 0.0006	± 0.32
V_{ud}	0.9734 ± 0.0008	± 0.42
B_e	(17.84 ± 0.06) %	± 1.82
$B_{\pi\pi^0}$	(25.41 ± 0.11) %	± 2.30
total		± 2.98

Largest error from $B_{\pi\pi^O}$

Systematic error (II)

Internal systematics

source		$\Delta a_\mu^{\pi\pi}$ (10^{-10})	comment
B.G. estimation			B.G. fraction
non-τ BG	**hadron**	± 0.05	0.14 ± 0.01 %
	2 photon	± 0.4	2.3 ± 0.06 %
Feed across BG	$h \geq 2\pi^0 \nu$	± 0.3	5.43 ± 0.08 %
	$K^- \pi^0$	± 1.2	1.74 ± 0.09 %
Energy scale		± 0.1	$\Delta E / E = \pm 0.2\%$ (π^0)
efficiency			
Minimum γ energy		± 1.8	80MeV – 200MeV
non-π^0 BG		± 0.8	Use π^0 side-band
total		± 2.36	

1. Non-τ BG.
   - estimated by B.G. MC.
   - control data sample are used for the calibration.
2. Feed-across
   - 1σ of measured Br.
3. Energy scale
   - uncertainty estimated from π^0 mass peak.
4. π^0 Selection
   - estimated from the uncertainty of side-band.
5. minimum γ energy

result

Result of $a_\mu^{\pi\pi}$ is …

preliminary

$$a_\mu^{\pi\pi} = \left(541.3 \pm 2.0(stat.) \pm 2.36(sys.) \pm 2.98(sys.\ ext.)\right) \times 10^{-10}$$

Integrated mass sqr. region : $4m_\pi^{\ 2}$ to $(1.8)^2 GeV^2$

cf. ALEPH

$$a_\mu^{\pi\pi} = \left(533.86 \pm 3.57(stat) \pm 2.36(sys)\right) \times 10^{-10}$$ (τ base)

Integrated mass sqr. region : $4m_\pi^{\ 2}$ to $(1.8)^2 GeV^2$

$$\frac{\left(a_\mu^{Belle} - a_\mu^{ALEPH}\right)}{\sqrt{\sigma_{Belle}^2 - \sigma_{ALEPH}^2}} = \frac{7.4}{4.7} = 1.6$$

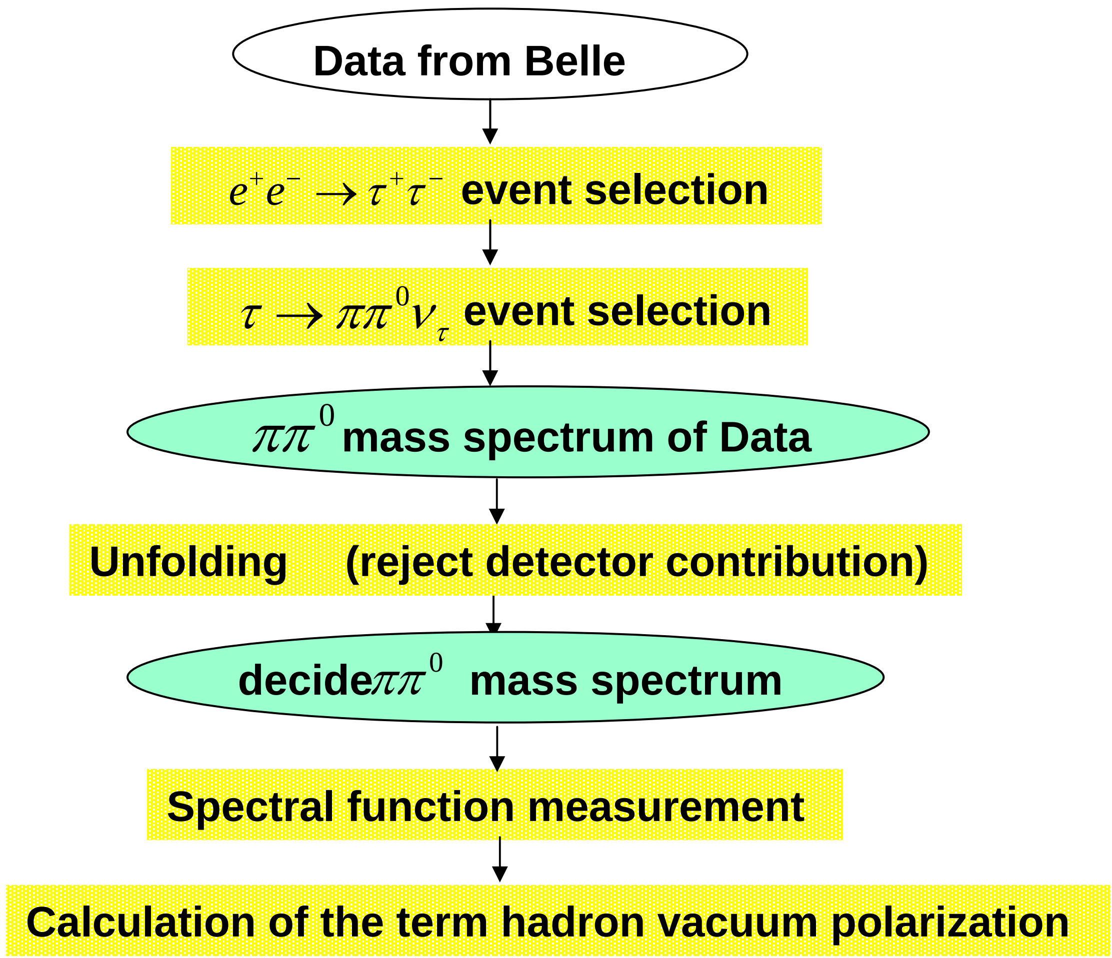
Consistent within error

excluding common error

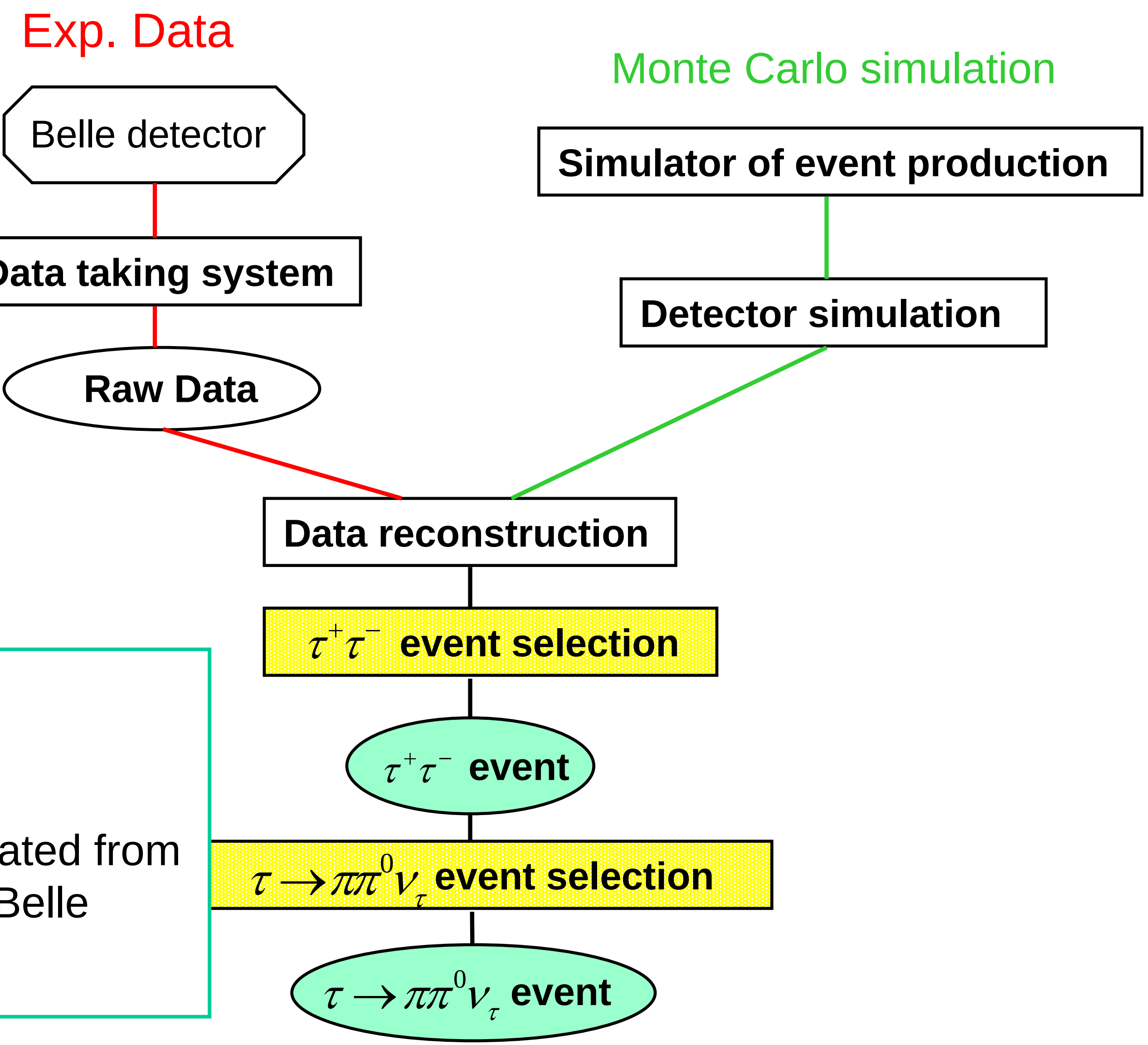
Backup slide

Analysis

Flow of this analysis



2.Event selection



Used Data

$4.44 fb^{-1}$ data accumulated from 2000.10 to 2000.12 at Belle

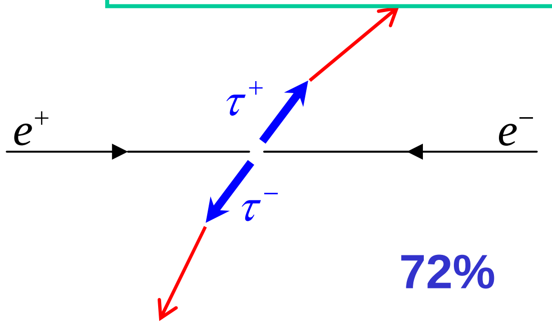
$\tau^+\tau^-$ data : 4.0×10^6

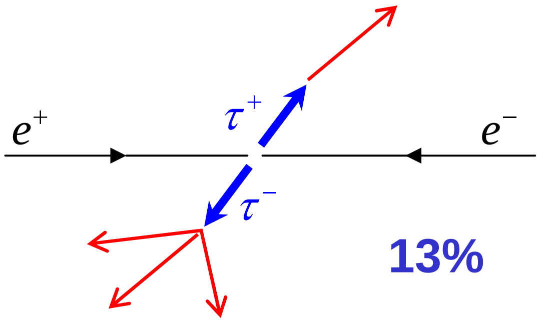
$e^+e^- \rightarrow \tau^+\tau^-$ event selection

τ decay

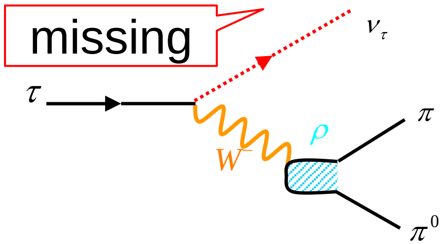
- There are few charged tracks in the event.

τ decay to 1 charged track ：	about 85%
τ decay to 3 charged tracks ：	about 15%





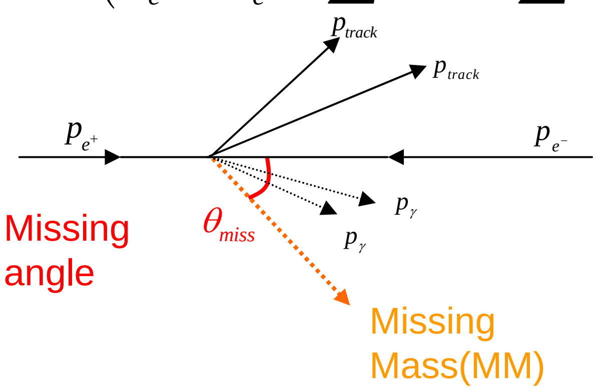
- There are missing of momentum and mass because of neutrino (ν) .

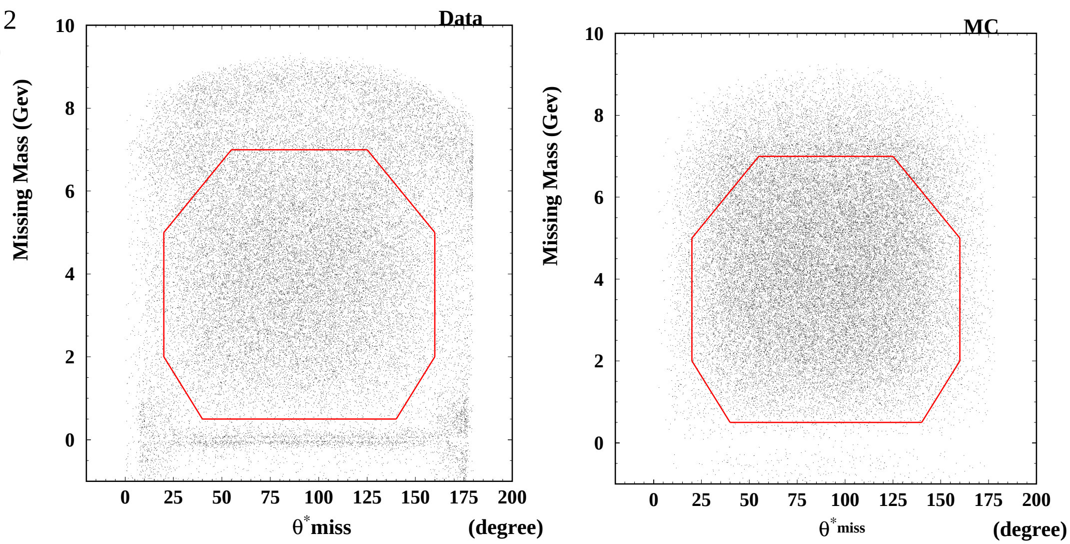


$e^+e^- \rightarrow \tau^+\tau^-$ event selection (Back ground rejection)

- Bhabha , $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and two photon rejection
  - clean Bhabha and mumu event rejection : $\sum|P| \leq 9.0\text{GeV/c}$, $\sum|E| \leq 9.0\text{GeV}$
  - Missing mass and Missing angle cut

$$MM^2 = \left(p_{e^+} + p_{e^-} - \sum p_{track} - \sum p_\gamma\right)^2$$

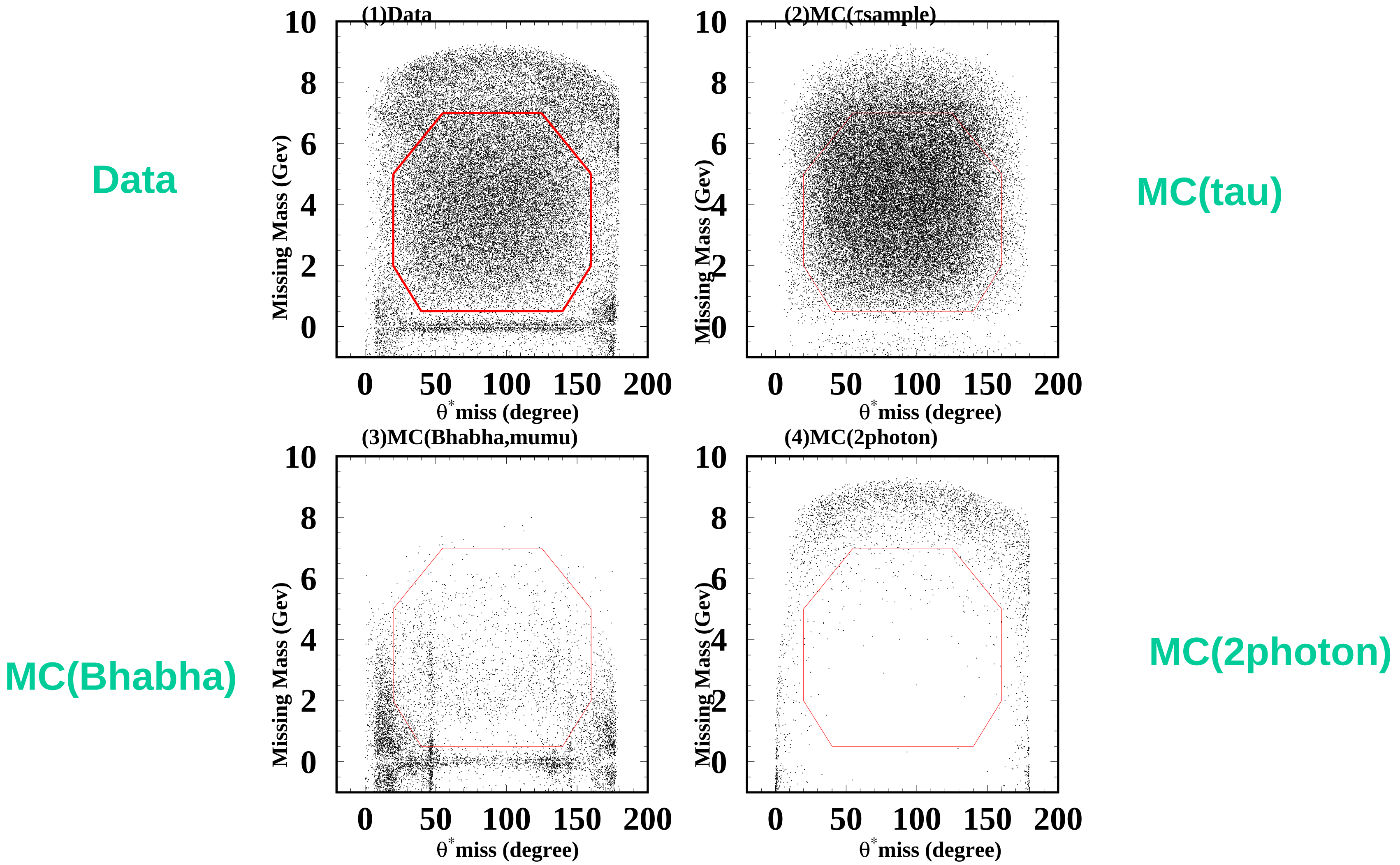




- Hadron($e^+e^- \rightarrow q\bar{q}$) rejection
  - Low-multiplicity event : $X_{part} \equiv (n_{track} + n_\gamma)_{one} \times (n_{track} + n_\gamma)_{other} \leq 25$

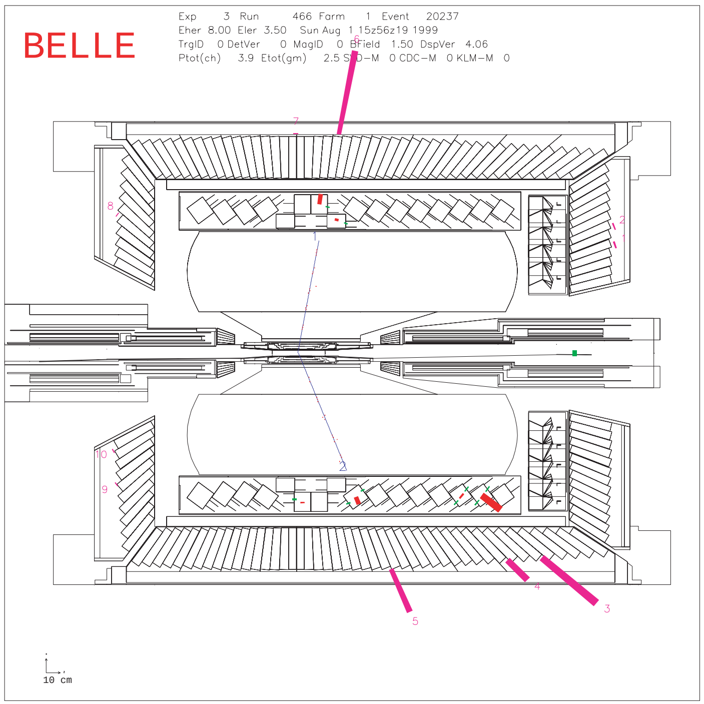
➡ **Then ,we obtained about 1,300,000 event of $e^+e^- \rightarrow \tau^+\tau^-$.**

Missing mass VS. Missing angle

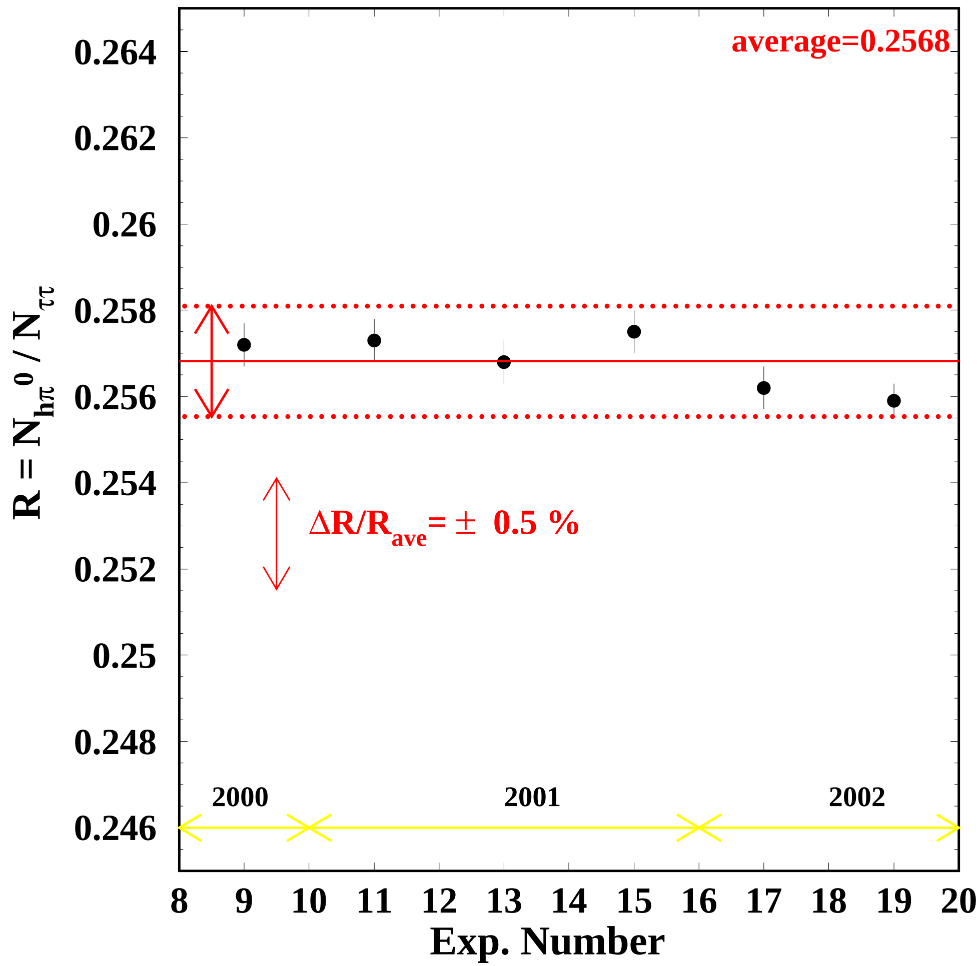


Time dependence

τ data at ***Belle*** detector



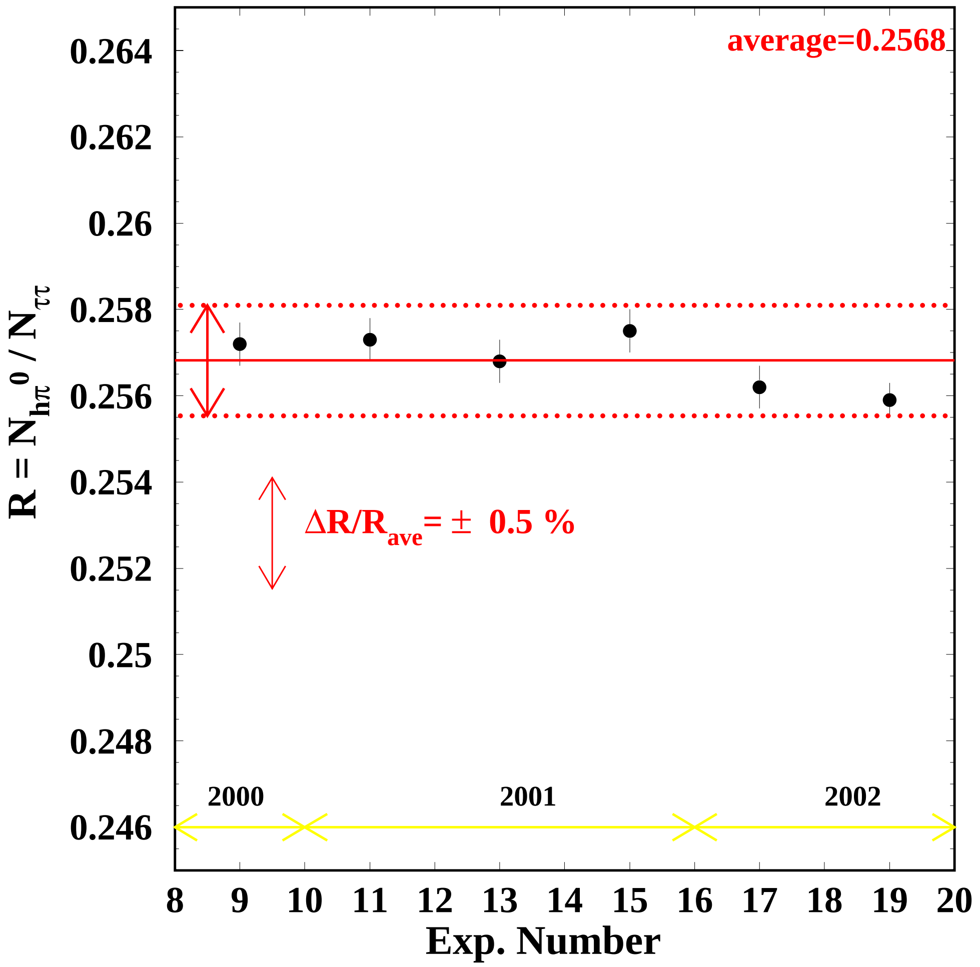
Time dependence of $\pi\pi^0$ event



$\dfrac{N_{\pi\pi^0}}{N_{\tau\tau}}$ is stable within 0.5%.

Time dependence

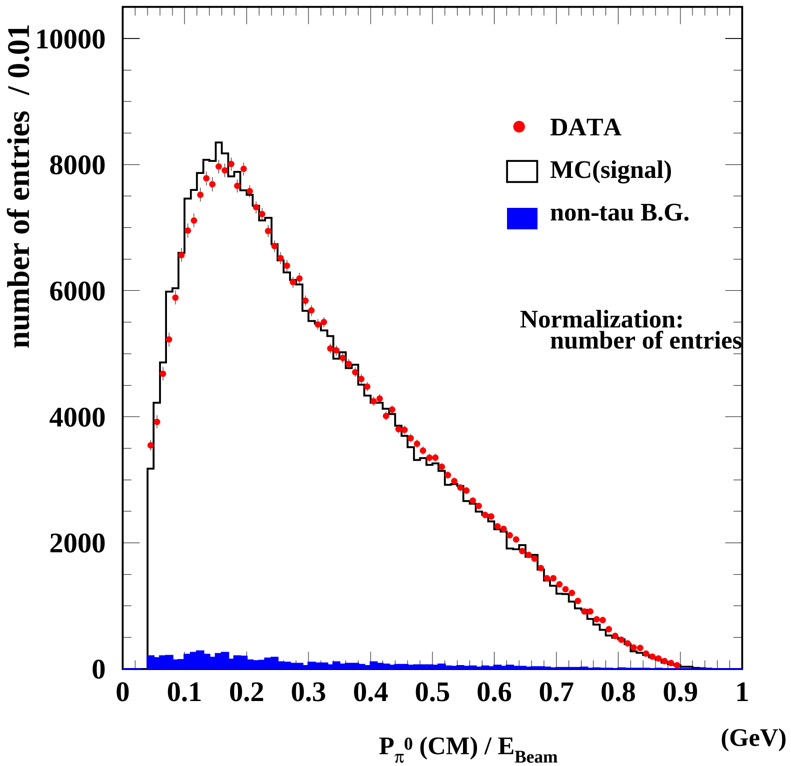
Time dependence of $\pi\pi^0$ event



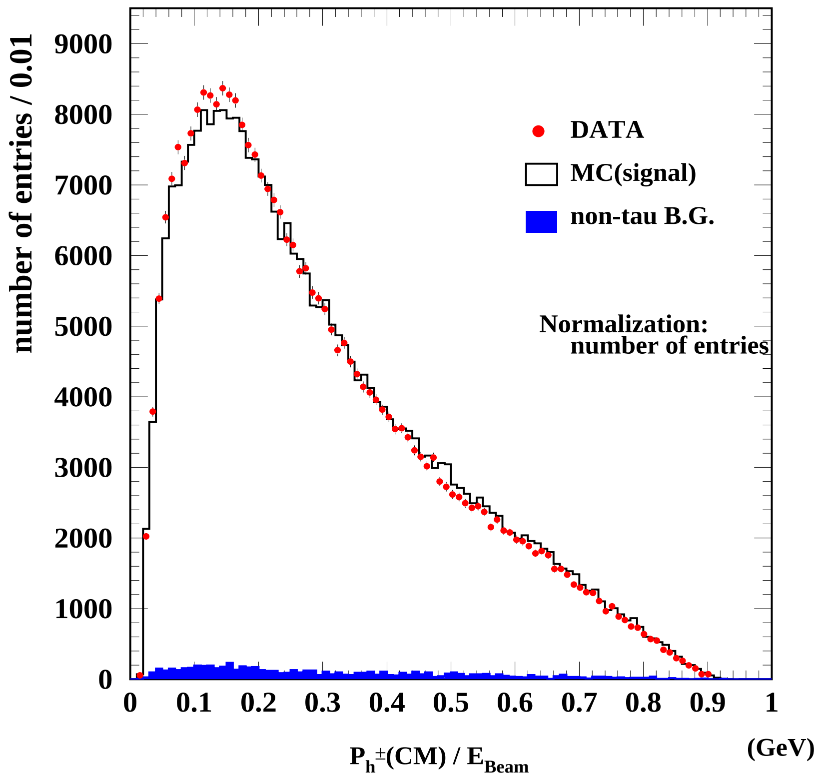
$$\frac{N_{\pi\pi^0}}{N_{\tau\tau}}$$ is stable within 0.5%.

Momentum of π^0 and $\pi^\pm$

π^0



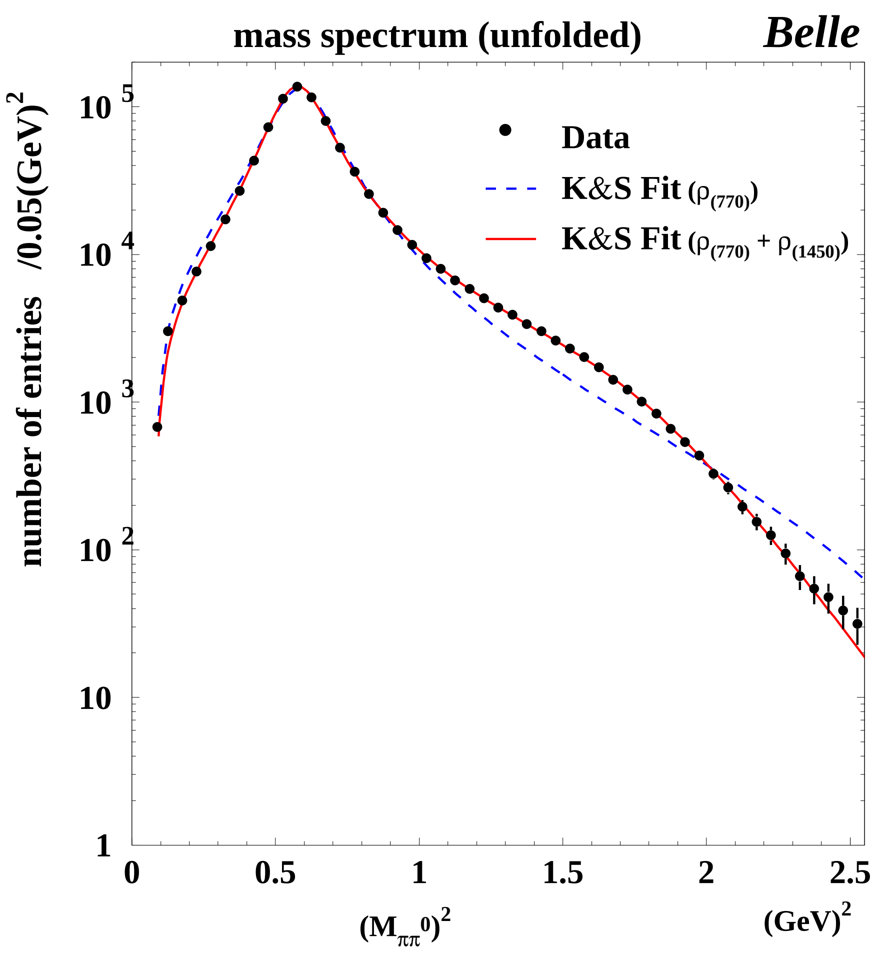
$\pi^\pm$



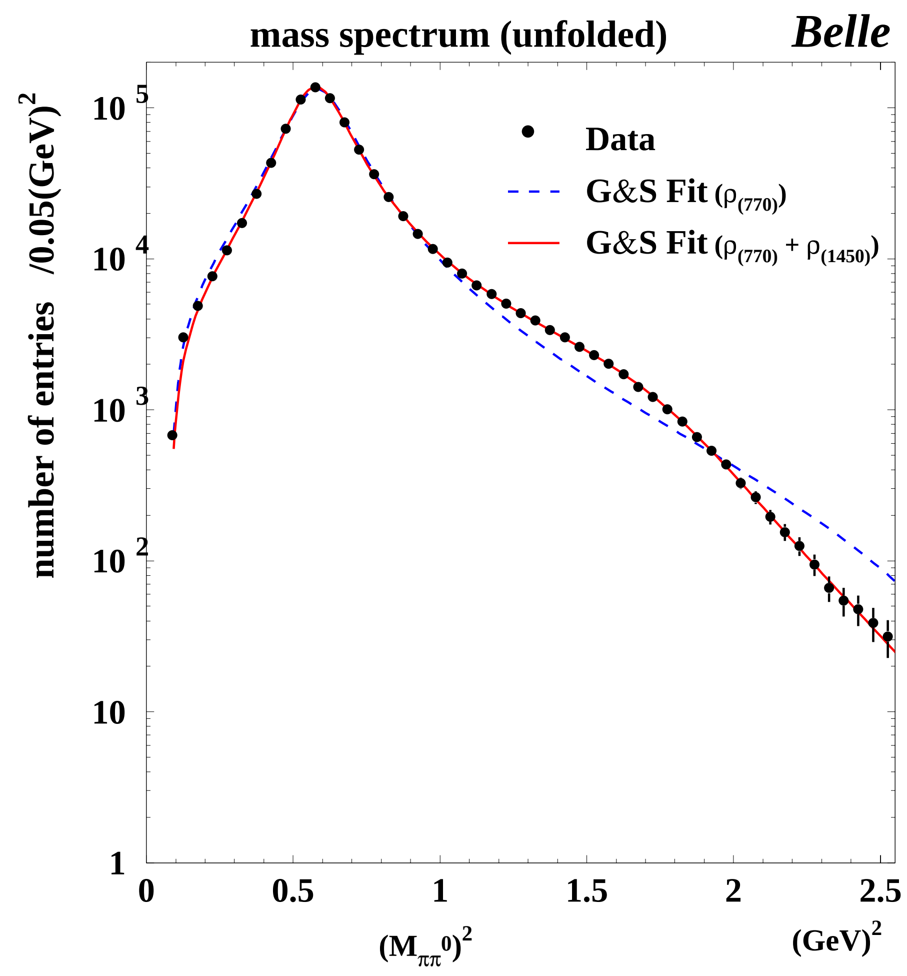
→ Good agreement between Data and MC .

Fitting result of Breit Wigner model

K&S model



G&S model



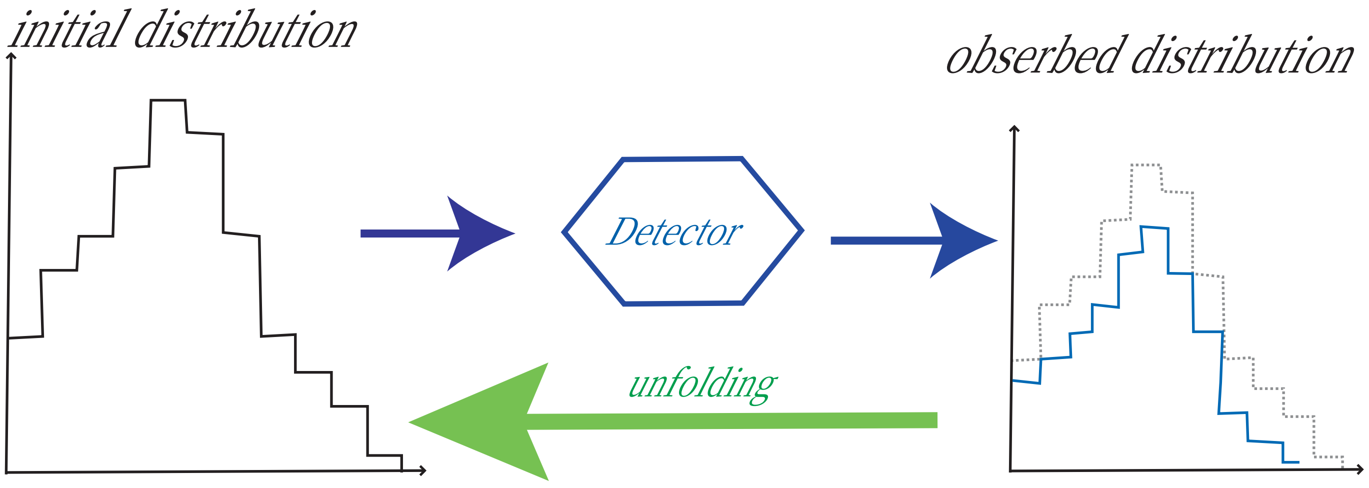
Fitting result

	K&S $\rho+\rho'$ (β is real)	K&S $\rho+\rho'$ (β,ϕ used)	G&S $\rho+\rho'$ (β is real)	G&S $\rho+\rho'$ (β,ϕ used)
M_ρ	773.25 ± 0.36	773.07 ± 0.39	773.94 ± 0.35	773.9 ± 0.37
Γ_ρ	150.58 ± 0.66	150.76 ± 0.68	152.37 ± 0.69	152.4 ± 0.71
$M_{\rho'}$	1397.8 ± 6.6	1421.7 ± 18.9	1395.0 ± 6.3	1398.2 ± 20.9
$\Gamma_{\rho'}$	514.77 ± 29.6	542.28 ± 41.5	445.9 ± 28.5	450.4 ± 39.9
β	-0.120 ± 0.005	0.14 ± 0.020	-0.084 ± 0.004	0.085 ± 0.010
ϕ		188.4 ± 9.05		180.0 ± 6.17
$\chi^2 / d.o.f$	$40.9/43 =$ 0.93	$38.8/42 =$ 0.93	$35.6/43 =$ 0.83	$35.6/42 =$ 0.85

Unfolding of invariant mass

About Unfolding

The observed distribution includes contribution of detector acceptance ,and smeared .



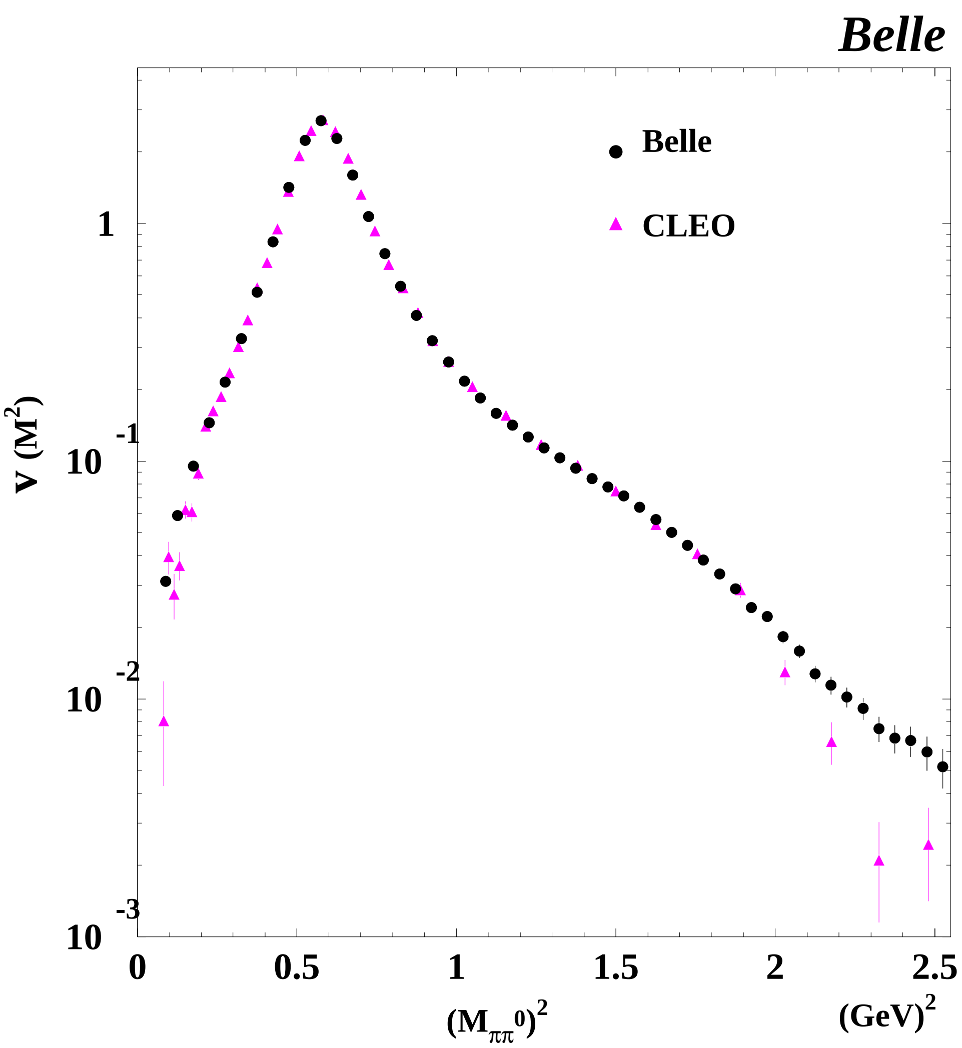
We can obtain true distribution by using Unfolding.

Unfolding is carried out by

Singular Value Decomposition (SVD) method.

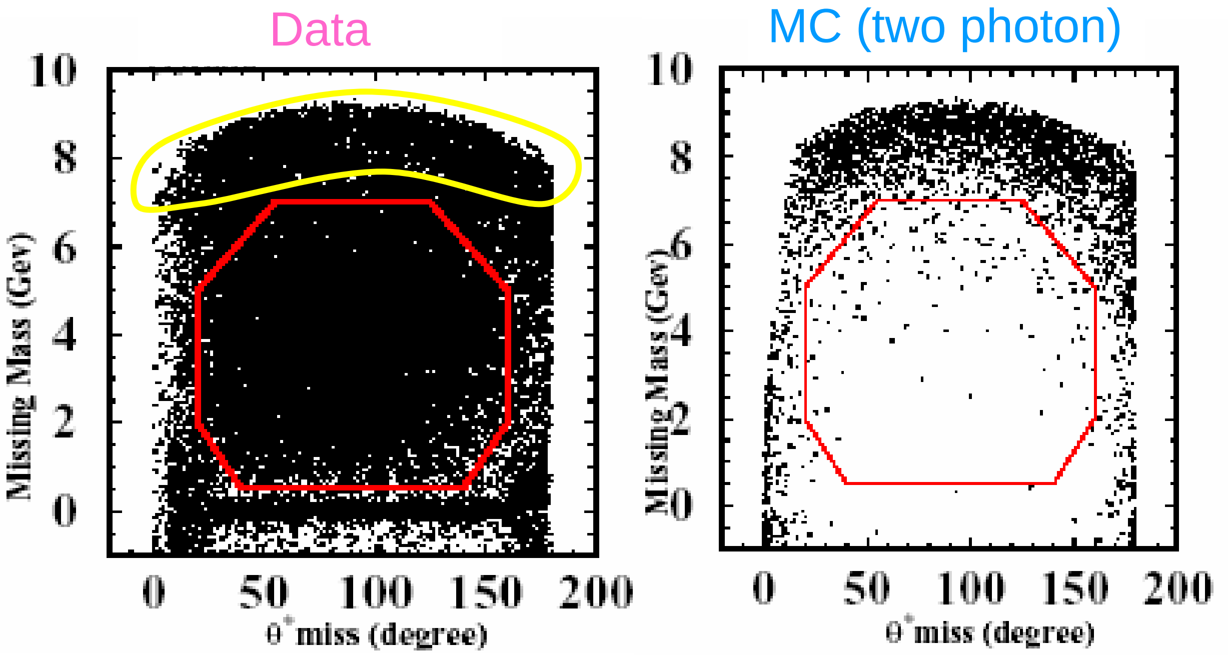
method a la ALEPH , A.Höcker, V.Karvelishvili,N.I.M. 372(1996)469

Spectral function



Systematic detail 1

(1) BG estimation (two photon)



Two photon B.G. estimated by Data using control sample.

(2) BG estimation (hadron)

Hadron BG contribution also estimated by Data using control sample.

$$X_{part} \equiv (n_{track} + n_{\gamma})_{one} \times (n_{track} + n_{\gamma})_{other} > 25 \qquad \text{for hadron selection}$$

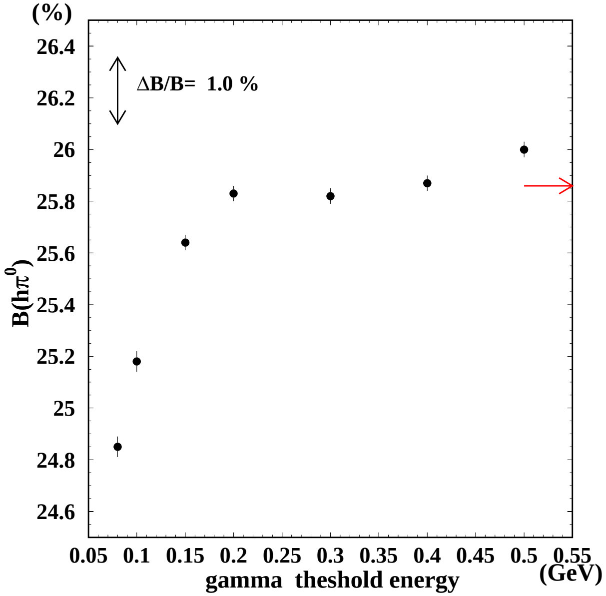
$$X_{part} \equiv (n_{track} + n_{\gamma})_{one} \times (n_{track} + n_{\gamma})_{other} \leq 25 \qquad \text{for } \tau \text{ selection}$$

Systematic detail2

(3) Energy scale

2% uncertainty of π^0 mass spectrum is assumed.

(4) Gamma energy threshold



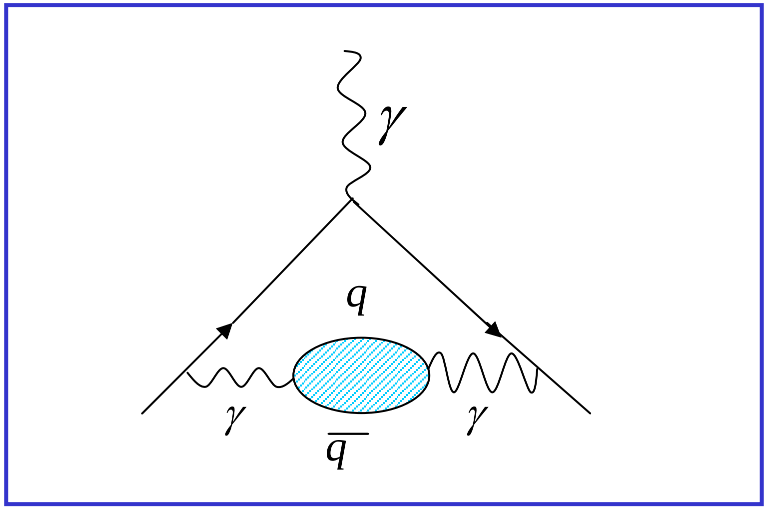
Uncertainty of $B(h\pi^0)$ as gamma-threshold function.

(5) π^0 side-band subtraction

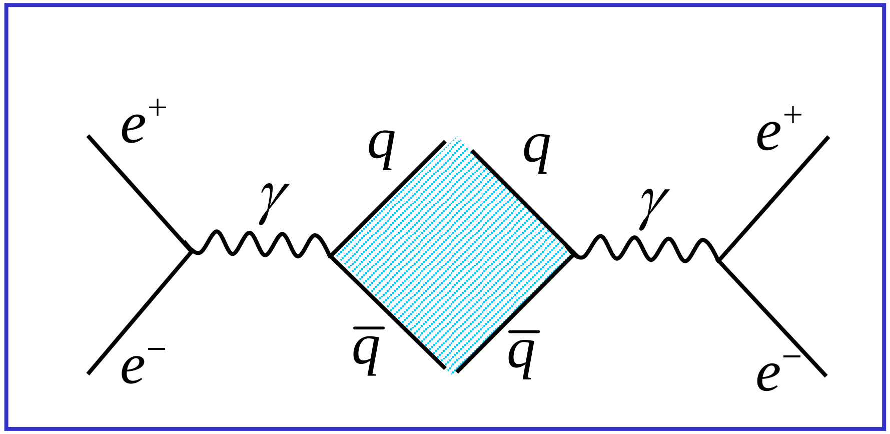
Use control sample of π^0 side-band .

Hadron Vacuum polarization and e^+e^- Data

The term of hadron vacuum polarization

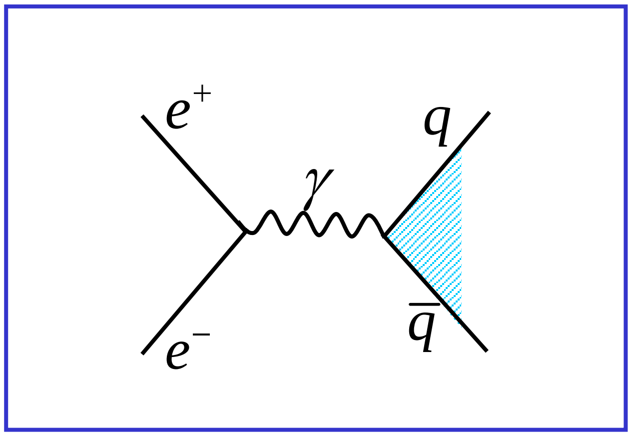


$e^+e^- \rightarrow hadron$

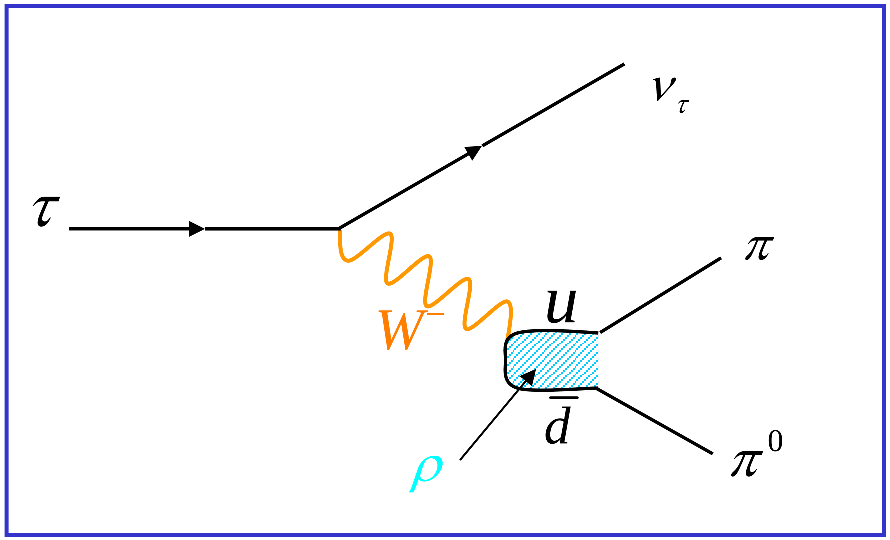


e^+e^- Data and τ Data

$e^+e^- \rightarrow hadron$



τ semi-Leptonic dacayc decay



Iso-spin、Cnserve of Vectro Current

↓

We can treat τ data as same condition as e^+e^-data.